Prescribed Performance Control of Uncertain MIMO Nonlinear Systems With Coupled and Constrained Inputs

Athanasios K. Gkesoulis, Haris E. Psillakis, George C. Karras, *Member, IEEE*, and Charalampos P. Bechlioulis, *Senior Member, IEEE*

***Abstract*— This letter addresses prescribed performance control (PPC) for uncertain multi-input multi-output (MIMO) nonlinear systems subject to saturated and coupled inputs. The proposed controller requires no model parameter knowledge and is of low implementation complexity, i.e., no regressors, observers, or approximation structures are utilized. We derive explicit feasibility conditions linking saturation levels to performance envelopes, system parameters, gains as well as disturbances, and effectively extend the low-complexity results on prescribed performance control with input saturation to the case of input coupling. Boundedness of all closed-loop signals and prescribed-performance tracking are established. A planar two-link manipulator simulation validates the approach.**

***Index Terms*— Robust adaptive control, constrained control, uncertain systems.**

I. Introduction

NONLINEAR systems with partially or completely unknown dynamics, arising from unmodeled nonlinearities, parametric uncertainty, or bounded disturbances, are ubiquitous in practice. For such systems, PPC [1] and funnel control (FC) [2] provide frameworks that guarantee strict transient and steady-state specifications, such as bounded overshoot, prescribed convergence rate, and settling-time targets, by encoding performance a priori through designer-chosen, time-varying envelopes. PPC and FC enforce these envelopes without detailed model knowledge, offering robustness against nonlinearities and disturbances.

In many practical systems, however, the presence of input constraints introduces significant challenges for achieving the desired control objectives. Actuator saturation due to physical limits, safety constraints, or energy restrictions can compromise tracking performance and even jeopardize stability if not properly addressed. The problem of controlling nonlinear systems under input limitations has therefore received long-standing research attention, leading to numerous adaptive and robust control methods designed to compensate for saturation effects [3], [4], [5], [6], [7]. To address input limitations within the prescribed performance framework, various PPC- and FC-based strategies have also been developed for uncertain nonlinear systems with input constraints. These include adaptive gain FC formulations for nonlinear [8], [9] and linear [10] systems, bang-bang FC controllers [11], [12], disturbance-regularity-based PPC formulations [13], saturation-tolerant PPC schemes for systems with bounded control gains, either employing fuzzy approximation and auxiliary dynamics to compensate for saturation effects [14], [15], or requiring large saturation levels to avoid saturation [16], designs that adaptively relax performance envelopes under saturation [17], [18], [19], [20], [21], [22], switching-based PPC approach [23] and PPC approaches that utilize neural or fuzzy approximation [17], [24], [25]. However, most of these methods either rely on model-dependent structures or employ complex approximation mechanisms such as neural networks and fuzzy schemes, complicating the implementation. Notably, only a few of them are of low implementation

Received 12 November 2025; revised 16 January 2026; accepted 9 February 2026. Date of publication 18 February 2026; date of current version 27 February 2026. This work was supported by the project "Applied Research for Autonomous Robotic Systems," implemented within the framework of the National Recovery and Resilience Plan "Greece 2.0" (Measure: 16618-Basic and Applied Research), funded by European Union–NextGenerationEU under Grant MIS 5200632. This article has not been presented at a conference. Recommended by Senior Editor P. Tesi. *(Corresponding author: Athanasios K. Gkesoulis.)*

Athanasios K. Gkesoulis is with the Robotics Institute, Athena Research Center, 151 25 Marousi, Greece (e-mail: a.gkesoulis@athenarc.gr).

Haris E. Psillakis is with the School of Electrical and Computer Engineering, National Technical University of Athens, 157 80 Athens, Greece (e-mail: hpsilakis@central.ntua.gr).

George C. Karras is with the Robotics Institute, Athena Research Center, 151 25 Marousi, Greece, and also with the Department of Informatics and Telecommunications, School of Sciences, University of Thessaly, 351 00 Lamia, Greece (e-mail: karrasg@athenarc.gr).

Charalampos P. Bechlioulis is with the Robotics Institute, Athena Research Center, 151 25 Marousi, Greece, and also with the Division of Systems and Automatic Control, Department of Electrical and Computer Engineering, University of Patras, 265 04 Patras, Greece (e-mail: chmpechl@athenarc.gr).

Digital Object Identifier 10.1109/LCSYS.2026.3666076

complexity [8], [9], [10], [11], [12], [18], [19], [20], [21], [22], [23], [26].

Among the aforementioned low-complexity PPC and FC designs for unknown nonlinear systems with input saturation, only [9], [19], [20], [22], [26] address MIMO systems. Relative degree one systems are considered in [9] and a two-funnel approach is proposed, where the outer funnel provides stability when saturation occurs. A two-funnel approach is also proposed in [22], where the outer funnel is dynamically adjusted to render input saturation feasible. In work [19], flexible adaptive performance envelopes are introduced, that degrade the performance requirements when the input saturates to ensure their feasibility under saturation, while in [20] and [26] reference modification mechanisms are utilized that properly modify the reference trajectory when the input saturates rendering the modified trajectory tracking problem feasible under input saturation. Among the aforementioned works, only [19], [22] consider arbitrary relative degrees and coupling between the inputs. However, the guaranteed tracking performance of [26] is better than those of [19] and [22]. Specifically, in [19] and [22] the tracking error, when the input saturates, is $O\left(\left|\frac{\Delta u}{e}\right|\right)$, where $\Delta u := \text{sat}(u) - u$ is the discrepancy between the saturated and requested control effort and $0 < |e| < 1$ in [19] and $0 < |e| \leq 1$ in [22], while in [26] the tracking error is $O(|\Delta u|)$. In work [20], the tracking error is in the classical prescribed performance control sense, with a theoretical lower bound for the input saturation level that coincides with the ones derived in [19] and [26]. However, in practice, the enhancement in performance guarantees is traded off with a larger required saturation level.

The consideration of coupled inputs is of practical and theoretical importance, as it naturally arises in mechanical and robotic systems, where actuator interactions are introduced through the system inertia matrix. Extending the low-complexity PPC schemes of [20] and [26] to the general MIMO case with input coupling is nontrivial. The principal difficulty stems from the fact that both the invariant set and the Lyapunov function constructions employed in [20] and [26] rely critically on decoupled input channels. When the inputs are coupled, these constructions cannot be directly extended, as the saturation of one input affects the evolution of all error channels, invalidating the decoupled stability properties and complicating feasibility analysis.

Motivated by these challenges, we develop a PPC design for a class of uncertain high-order MIMO nonlinear systems with input coupling and saturation. To achieve this, we utilize a filtered tracking error with an added modification term, that properly alters the error whenever saturation occurs, to ensure system stability. To overcome the technical difficulties of extending the results of [20] and [26] to uncertain MIMO nonlinear systems with coupled inputs, we introduce a new invariant set formulation and a novel Lyapunov-like function that jointly handle coupled input dynamics while preserving the prescribed performance guarantees. These generalizations render the proposed method both theoretically significant and practically applicable to complex nonlinear systems.

The main contributions of this work can be summarized follows:

- We extend low-complexity PPC designs based on reference modification [20], [26] to general MIMO nonlinear systems with input coupling. We overcome the technical difficulty of applying the analysis of [20] and [26] to coupled-input systems by introducing a new theoretical approach to the analysis of prescribed performance control for MIMO systems with input coupling and saturation.
- We achieve an $O(|\Delta u|)$ bound for the tracking error, when saturation occurs, improving the related low-complexity works [19], [22] for MIMO nonlinear systems with input coupling and saturation.
- We develop a low-complexity design, i.e. no observers, neural networks or fuzzy structures and no knowledge of system parameters are required. Filtering of the tracking error is utilized, which further decreases the design and analysis complexity. We provide explicit, closed-form lower bounds on input saturation levels that highlight their connection to the performance envelopes, closed-loop gains, system dynamics and disturbances.

The rest of the letter is organized as follows. Section II states the problem, modeling assumptions, and performance objective. Section III presents the controller and the stability and performance analysis. Section IV provides simulations validating the theoretical results.

II. Problem Formulation

A. System Dynamics

Consider the r-th order MIMO nonlinear system with m inputs and m outputs of relative degree $\{r_1, \ldots, r_m\}$, respectively, with dynamics:

$$\begin{aligned}\dot{x}_i^j &= x_i^{j+1}, \quad j = 1, \ldots, r_i - 1,\\ \dot{x}_i^{r_i} &= f_i(t,x) + g_i(t,x)\text{sat}(u),\\ y_i &= x_i^1, \quad i = 1, \ldots, m,\end{aligned} \tag{1}$$

where $x := [x_1^T, \ldots, x_m^T]^T \in \mathbb{R}^r$ denotes the state vector, with $x_i := [x_i^1, \ldots, x_i^{r_i}]^T \in \mathbb{R}^{r_i}$ and $r := r_1 + \cdots + r_m$, $u := [u_1, \ldots, u_m]^T$ is the unconstrained control input and $y := [x_1^1, \ldots, x_m^1]^T \in \mathbb{R}^m$ is the output. The functions $f_i : \mathbb{R}^+ \times \mathbb{R}^r \to \mathbb{R}$, $g_i : \mathbb{R}^+ \times \mathbb{R}^r \to \mathbb{R}^{1\times m}$, incorporate unknown nonlinearities and bounded disturbances, and can be written as $f := [f_1(t,x), \ldots, f_m(t,x)]^T \in \mathbb{R}^m$ and $G := [g_1^T(t,x) \ldots g_m^T(t,x)]^T \in \mathbb{R}^{m\times m}$. The control input is subject to saturation and we denote the saturated control input as $\text{sat}(u) := [\text{sat}(u_1) \cdots \text{sat}(u_m)]^T \in \mathbb{R}^m$, where the saturation function is defined as:

$$\text{sat}(u_i) := \begin{cases} u_i, & |u_i| < u_{i,\max},\\ u_{i,\max}\text{sgn}(u_i), & \text{otherwise},\end{cases} \tag{2}$$

with saturation levels $u_{i,\max} > 0$, $i = 1, \ldots, m$. Let the desired trajectory be $y_d(t) := [y_{d,1}(t), \ldots, y_{d,m}(t)]^T \in \mathbb{R}^m$, and define the reference vector $x_{d,i}(t) = [y_{d,i}(t), \ldots, y_{d,i}^{(r_i-1)}(t)]^T \in \mathbb{R}^{r_i}$, which contains the desired trajectory and its first $r_i - 1$ derivatives. We also make the following assumptions:

Assumption 1: The vector function f and matrix G are continuous functions with respect to (w.r.t.) time and locally Lipschitz w.r.t. to x.

Assumption 2: The vector function f and matrix G are uniformly bounded w.r.t. time to incorporate the effect of time-dependent bounded disturbances.

Assumption 3: $G(t, x)$ is a uniformly positive definite matrix for all $(t, x) \in \mathbb{R}^+ \times \mathbb{R}^r$.

Assumption 4: The reference trajectories $y_{d,i}(t), i = 1, \ldots, m$, are known uniformly bounded functions of time with bounded known derivatives up to order $r_i - 1$. The bounds are not employed in the controller design.

B. Control Problem Definition

Prescribed performance refers to enforcing the evolution of the tracking error strictly within a time-varying envelope that encodes desired transient and steady-state characteristics. Specifically, let $e_i(t)$ denote a measurable tracking error, for each $i = 1, \ldots, m$. Prescribed performance is achieved if $|e_i(t)| < \rho_i(t)$, for all $t \geq 0$ and $i = 1, \ldots, m$, where $\rho_i(t)$ is a monotonically decreasing performance function of the form:

$$\rho_i(t) = (\rho_i^0 - \rho_i^\infty)e^{-l_i t} + \rho_i^\infty, \quad \rho_{i,0}, \rho_{i,\infty}, l_i > 0, \tag{3}$$

with ρ_i^0, ρ_i^∞, and l_i denoting the initial and steady-state bounds and decay rate, respectively. Such envelope-based error regulation is closely aligned with frameworks that enforce coordinated convergence of all error components within pre-defined time profiles [27], [28].

In the presence of saturation, enforcing classic PPC directly on the plant often amounts to implicitly letting $u_{i,\max}$ tend to infinity, defeating the purpose of constraints [26]. Our goal is to maintain performance envelopes while acknowledging hard input limitations. Hence, we aim at designing a continuous state-feedback control law u that:

1) ensures prescribed performance tracking whenever saturation limits allow,
2) guarantees boundedness of all closed-loop signals,
3) requires no knowledge of the system vector f and matrix G, and
4) is of low-complexity in the sense that no observers, or approximation structures such as neural networks or fuzzy systems are utilized.

III. Main Results

A. Control Design

The control design can be divided in the following steps:

Step 1: For each instance $i = 1, \ldots, m$, consider the filtered errors:

$$s_i(\tilde{e}_i(t)) := q_i^T \tilde{e}_i(t) = \left[\prod_{j=1}^{r_i-1}\left(\frac{d}{dt} + p_i^j\right)\right] e_i(t), \tag{4}$$

where $\tilde{e}_i(t) := [\tilde{e}_i^1(t) \cdots \tilde{e}_i^{r_i}(t)]^T = x_i(t) - x_{d,i}(t)$, $e_i(t) := y_i(t) - y_{d,i}(t) = [\tilde{e}_1^1 \cdots \tilde{e}_m^1]^T$, and $q_i := [q_i^1, q_i^2, \ldots, q_i^{r_i-1}, 1]^T \in \mathbb{R}^{r_i}$. The constants $p_i^j > 0$ are selected so that the polynomial $s^{r_i-1} + q_i^{r_i-1}s^{r_i-2} + \cdots + q_i^2 s + q_i^1$ is Hurwitz, with $\min_{1\leq j\leq r_i-1}\{p_i^j\} > l_i$, $i = 1, \ldots, m$.

Step 2: Design the modification signal $\sigma(t) := [\sigma_1(t) \cdots \sigma_m(t)]^T$ to be generated dynamically by:

$$\dot{\sigma}(t) = -\beta\sigma(t) + \Delta u(t), \tag{5}$$

where $\sigma(0) = 0$, $\beta > 0$ is a design gain and $\Delta u(t) := [\Delta u_1(t) \cdots \Delta u_m(t)]^T = \mathrm{sat}(u(t)) - u(t)$. The signal $\sigma(t)$ directly tracks the mismatch between the required and the actual control effort. When no saturation occurs, $\Delta u = 0$ and the signal decays exponentially to zero.

Step 3: Define the normalized error vector $\xi := [\xi_1 \cdots \xi_m]^T$, where:

$$\xi_i(t) := \frac{s_i(\tilde{e}_i(t)) - \sigma_i(t)}{\rho_i(t)}, \tag{6}$$

where ρ_i are of the form (3), with $\rho_i^0 > |s_i(\tilde{e}_i(0))|$, for all $i = 1, \ldots, m$. The control law is then designed as:

$$u_i(t) := -k_i T(\xi_i), \tag{7}$$

where k_i are positive design constants and $T : (-1, 1) \to \mathbb{R}$ is a strictly increasing, continuously differentiable function, satisfying $\lim_{x\to 1^-} T(x) = \infty$, and $\lim_{x\to -1^+} T(x) = -\infty$, e.g., $T(\cdot) = \ln\frac{1+(\cdot)}{1-(\cdot)}$. The normalized error vector ξ injects the modification signal $\sigma(t)$ into the filtered error, effectively altering it to render tracking with prescribed performance feasible under input saturation. The control law can be written in vector form as:

$$u(t) := -K\mathbf{T}(\xi), \tag{8}$$

where $K := \mathrm{diag}(k_1, \ldots, k_m)$ and $\mathbf{T}(\xi) := \left[T(\xi_1) \cdots T(\xi_m)\right]^T$.

Remark 1: The preceding control design does not use any a priori knowledge of the system matrices and vectors and therefore the system dynamics are considered unknown to the controller.

B. Stability and Performance Analysis

Let us state here a preliminary Lemma:

Lemma 1: [[19]] Consider the vector $\tilde{e}_i(t) \in \mathbb{R}^m$ as well as the filtered error defined in (4) and the decaying performance function (3) with $l_i < \min_{j=1,\ldots,r_i-1}\{p_i^j\}$, $i = 1, \ldots, m$. If $|s_i(\tilde{e}_i(t))| < \rho_i(t)$, $\forall t \geq 0$,then for each element $\tilde{e}_i^j(t)$, $j = 1, \ldots, r_i$ of the tracking error $\tilde{e}_i(t)$ there exist constants $\bar{e}_i^j > 0$, such that:

$$\left|\tilde{e}_i^j(t)\right| \leq \bar{e}_i^j \exp(-l_i t) + \frac{2^{j-1}\rho_{i,\infty}}{\prod_{k=1}^{r_i-j} p_i^k}, \quad \forall t \geq 0. \tag{9}$$

We are now ready to state the main result of this letter:

Theorem 1: Consider the r-th order MIMO nonlinear system (1) under Assumptions 1–4. Let the control law (8) be applied. If, for some constants $\Lambda_i > 0$, the input saturation limits satisfy:

$$u_{i,\max} \geq \max\Bigg\{k_i T\big(|\xi_i(0)|\big) - \Lambda_i, \sup_{t\in[0,\infty)} \max_{x\in S} \frac{\|H(t,x)\| + \sqrt{m}\left(\|\dot{R}(t)\| + \Lambda_{\max}\right)}{\lambda_{\min}(G(t,\xi))}\Bigg\}, \tag{10}$$

where $H(t, x) := [h_1(t, x) \cdots h_m(t, x)]^T$, with $h_i(t, x) := \sum_{j=1}^{r_i-1} q_i^j \tilde{e}_i^{j+1} - y_{d,i}^{(r_i)}(t) + f_i(t, x)$, $R(t) := \mathrm{diag}(\rho_1(t), \ldots, \rho_m(t))$, $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue, $S := \{x \in \mathbb{R}^r \| |\tilde{e}_i^j(t)| \leq \hat{e}_i^j, \forall j = 1, \ldots, r_i, i = 1, \ldots, m\}$ are compact sets with $\hat{e}_i^j := \bar{e}_i^j + \frac{2^{j-1}(\rho_i^\infty + (\Lambda_{\max}/\beta))}{\prod_{k=1}^{r_i-j} p_i^k}$, and $\Lambda_{\max} := \max_{1\leq j\leq m} \Lambda_j$, where

$\bar{e}_i^j > 0$ depend on initial conditions and the parameters of the prescribed performance functions, for all $i = 1, \ldots, m$. Then:

1) All closed-loop signals remain bounded for all $t \geq 0$.
2) The prescribed performance specifications are satisfied in the sense that:

$$|\tilde{e}_i^j(t)| \leq \bar{e}_i^j e^{-l_i t} + \frac{2^{j-1}\rho_i^\infty}{\prod_{k=1}^{r_i-j} p_i^k} + \frac{2^{j-1}\sup_{t\geq 0}|\Delta u_i|}{\beta\prod_{k=1}^{r_i-j} p_i^k}, \quad (11)$$

and:

$$|\sigma_i(t)| \leq \frac{\sup_{t\geq 0}|\Delta u_i|}{\beta}, \quad (12)$$

for all $t \geq 0$ and every $i = 1, \ldots, m$ and all $t \geq 0$.

Proof: Differentiating (6), we obtain that:

$$\dot{\xi}_i = \frac{1}{\rho_i(t)}\big(h_i(t,x) + g_i(t,x)\mathrm{sat}(u) + \beta\sigma_i - \Delta u_i - \dot{\rho}_i(t)\xi_i\big).$$

Thus, we can write $\xi = [\xi_1 \cdots \xi_m]^T \in \mathbb{R}^m$ in vector form as:

$$\dot{\xi} = R^{-1}(t)\Big(H(t,x) + G(t,x)\mathrm{sat}(u) + \beta\sigma - \Delta u - \dot{R}(t)\xi\Big). \quad (13)$$

Let us now define the augmented state vector, $\xi_{ag} := \left[\tilde{e}_{r,1}^T \cdots \tilde{e}_{r,m}^T\ \xi^T\ \sigma^T\right]^T \in \mathbb{R}^{r+m}$, where $\tilde{e}_{r,i} := [\tilde{e}_i^1 \cdots \tilde{e}_i^{r_i-1}]^T$. Invoking (1), (4), (5), (8) and (13) we obtain that:

$$\dot{\xi}_{ag} = \Phi\left(t, \xi_{ag}\right) \quad (14)$$

Consider now the set $\Omega = \mathbb{R}^{r-m} \times (-1,1)^m \times \mathbb{R}^m$. The function Φ is continuous w.r.t. time and locally Lipschitz w.r.t. ξ_{ag} in Ω. It holds that $|\xi_i(0)| < 1$, for all $i = 1, \ldots, m$, by the appropriate selection of parameters ρ_i^0 in Section III-A. Therefore, $\xi_{ag}(0) \in \Omega$ and (14) admits a unique maximal solution $\xi_{ag} : [0, t_{\max}) \to \Omega$, with $t_{\max} \in (0, \infty]$.

Since $T(\xi_i(0))$ is bounded by construction, for every selection of $\Lambda_i > \max\{0, |u_i(0)| - u_{i,\max}\}$, $i = 1, \ldots, m$, there exists a time subinterval $[0, t_s) \subseteq [0, t_{\max})$, with $t_s > 0$, such that:

$$|u_i(t)| \leq \Lambda_i + u_{i,\max} \quad (15)$$

and therefore:

$$|T(\xi_i(t))| \leq \frac{\Lambda_i + u_{i,\max}}{k_i}, \quad (16)$$

$$|\xi_i(t)| \leq T^{-1}\left(\frac{\Lambda_i + u_{i,\max}}{k_i}\right) =: \bar{\xi}_i, \quad (17)$$

and:

$$|\Delta u_i(t)| \leq \Lambda_i, \quad (18)$$

for all $t \in [0, t_s)$ and $i = 1, \ldots, m$. Combining (18) and (5) we obtain, for all $i = 1, \ldots, m$, that:

$$|\sigma_i(t)| < \bar{\sigma}_i := \frac{\Lambda_i}{\beta}, \ \forall t \in [0, t_s). \quad (19)$$

Invoking (6), we obtain, for all $i = 1, \ldots, m$, that $s_i(\tilde{e}_i(t)) = \rho_i(t)\xi_i(t) + \sigma_i(t)$, which, by virtue of Assumption 4 and Lemma 1, implies that $|\tilde{e}_i^j(t)| \leq \hat{e}_i^j$, for all $t \in [0, t_s)$, $j = 1, \ldots, r_i$ and $i = 1, \ldots, m$.

Let us now define the set $\mathcal{T} := \cap_{i=1}^m \mathcal{T}_i$, where $\mathcal{T}_i := \left\{\xi \in \mathbb{R}^m | T(|\xi_i|) \leq \frac{u_{i,\max}+\Lambda_i}{k_i}\right\}$, for all $i = 1, \ldots, m$. The next part of the proof is dedicated to extracting a feasibility condition for $u_{i,\max}$ that will render the set $\mathcal{T}$ positively invariant and guarantee that $\xi(0) \in \mathcal{T}$. To prove the positive invariance, we distinguish the following cases:

Case 1: If $\xi \in \mathrm{int}\mathcal{T}$, then from the definition of $\mathcal{T}$ we obtain that:

$$T(|\xi_i|) < \frac{u_{i,\max} + \Lambda_i}{k_i},$$

for each $i = 1, \ldots, m$ and since T are strictly increasing functions:

$$|\xi_i(t)| < T^{-1}\left(\frac{u_{i,\max} + \Lambda_i}{k_i}\right) \quad (20)$$

and as a result (15)-(19) hold, for each $i = 1, \ldots, m$.

Case 2: If $\xi \in \partial\mathcal{T}$, let us define:

$$V(t,\xi) := \sum_{i=1}^m \rho_i(t) F_i(\xi_i), \quad (21)$$

where $F_i(\xi_i) := \int_0^{\xi_i} \mathrm{sat}(k_i T(s))\, ds$ are positive definite and radially unbounded functions for all $i = 1, \ldots, m$. Taking the derivative of V w.r.t. time, we obtain:

$$\dot{V} = \sum_{i=1}^m \rho_i(t)\mathrm{sat}(k_i T(\xi_i))\dot{\xi}_i + \sum_{i=1}^m \dot{\rho}_i(t) F_i(\xi_i). \quad (22)$$

Noting that ρ_i are strictly decreasing, we have that:

$$\dot{V} \leq \mathrm{sat}^T(K_v \mathbf{T}(\xi))\big(H(t,x) - G(t,\xi)\mathrm{sat}(K_v\mathbf{T}(\xi)) + \beta\sigma - \Delta u - \dot{R}(t)\xi\big), \quad (23)$$

where $\mathrm{sat}(K_v\mathbf{T}(\xi)) := \left[\mathrm{sat}(k_1 T(\xi_1)) \cdots \mathrm{sat}(k_m T(\xi_m))\right]^T$. Since $\mathrm{sgn}(\Delta u_i) = \mathrm{sgn}(k_i T(\xi_i))$ and $\xi \in \partial\mathcal{T}$, substituting from (19), (18) and the Rayleigh inequality, we obtain:

$$\dot{V} \leq -\lambda_{\min}(G(t,\xi))\|\mathrm{sat}(K_v\mathbf{T}(\xi))\|^2 + \|\mathrm{sat}(K_v\mathbf{T}(\xi))\|\left(\|H(t,x)\| + \sqrt{m}\left(\|\dot{R}(t)\| + \Lambda_{\max}\right)\right). \quad (24)$$

Due to $\xi \in \partial\mathcal{T}$, we have that $T(|\xi_i|) = \frac{u_{i,\max}+\Lambda_i}{k_i}$ for at least one $i \in \{1, \ldots, m\}$ and therefore, for that i, it holds that $\mathrm{sat}(k_i T(|\xi_i|)) = u_{i,\max}$. Utilizing this and the properties of the norm, it holds true that:

$$\|\mathrm{sat}(K_v\mathbf{T}(\xi))\| \geq \max_{1\leq i\leq m}|\mathrm{sat}(k_i T(\xi_i))| \geq \min_{1\leq i\leq m} u_{i,\max}.$$

Substituting to (24), we have that:

$$\dot{V} \leq -\|\mathrm{sat}(K_v\mathbf{T}(\xi))\|\Bigg(\lambda_{\min}(G(t,\xi))\min_{1\leq i\leq m} u_{i,\max} - \|H(t,x)\| - \sqrt{m}\left(\|\dot{R}(t)\| + \Lambda_{\max}\right)\Bigg). \quad (25)$$

Therefore, a sufficient condition that guarantees that $\dot{V}$ is non-positive on the boundary of $\mathcal{T}$ is:

$$\min_{1\leq i\leq m} u_{i,\max} \geq \sup_{t\in[0,t_s)}\max_{x\in S}\frac{\|H(t,x)\| + \sqrt{m}\left(\|\dot{R}(t)\| + \Lambda_{\max}\right)}{\lambda_{\min}(G(t,\xi))}. \quad (26)$$

We now proceed to establish a sufficient condition that guarantees that $\xi(0) \in \mathcal{T}$, which is equivalent to:

$$T(|\xi_i(0)|) \leq \frac{u_{i,\max} + \Lambda_i}{k_i}, \quad (27)$$

for all $i = 1, \ldots, m$ and to:

$$|\xi_i(0)| \leq T^{-1}\left(\frac{u_{i,\max} + \Lambda_i}{k_i}\right), \tag{28}$$

for all $i = 1, \ldots, m$. As a result the sufficient condition is $u_{i,\max} \geq k_i T(|\xi_i(0)|) - \Lambda_i$, for all $i = 1, \ldots, m$.

Therefore, if (10) holds for all $i = 1, \ldots, m$, then $\xi(0) \in \mathcal{T}$, and $\mathcal{T}$ is a positively invariant set. By a contradiction argument, it follows that $t_s = t_{\max}$ and $\xi(t) \in \mathcal{T}$, for all $t \in [0, t_{\max})$. Consequently, we obtain that $|\xi_i(t)| \leq T^{-1}\left(\frac{u_{i,\max}+\Lambda_i}{k_i}\right)$ (see (20) for the interior of $\mathcal{T}$) and thus $|\xi_i(t)| \in \left[-\bar{\xi}_i, \bar{\xi}_i\right] \subset (-1, 1)$, for every $i = 1, \ldots, m$ and all $t \in [0, t_{\max})$. Using standard arguments, we conclude that $t_{\max} = \infty$ and all closed-loop signals remain bounded. Finally, since $|\xi_i(t)| < 1$, and because u_i is bounded for all $t \geq 0$, invoking (6), (5) and Lemma 1 we conclude the theorem. ■

Remark 2: The Lyapunov-like function (21) enables the extension of the stability and performance analysis results of [26] to the coupled input case, while considering a multi-dimensional version $\mathcal{T}$ of the one-dimensional invariant sets $\mathcal{T}_i$. This way $O(|\Delta u_i|)$ bounds for the tracking errors are established, when saturation occurs in dimension i, providing improved bounds compared to the related low-complexity PPC work [19].

Remark 3: The feasibility condition presented in Theorem 1 is sufficient for solving the control problem. However, this feasibility condition is not necessary for solving the aforementioned problem, because the maximum and supremum operators involved in the Theorem's formulation are conservative and reflect the worst-case interaction between system dynamics and the required control input. In practice, a much lower values for $u_{i,\max}$ is likely to be adequate, as stated in [26].

Remark 4: Since the system dynamics are unknown, the quantities $\|H(t, x)\|$ and $\lambda_{\min}(G(t, x))$ cannot be determined a priori. Thus, (10) serves as a structural feasibility characterization rather than a numerically computable expression. Nevertheless, it explicitly shows that the required saturation levels $u_{i,\max}$ grow with $\|H(t, x)\|$ and $\|\dot{R}(t)\|$, and decrease with $\lambda_{\min}(G(t, x))$. In this way, it clarifies how the prescribed performance specifications, disturbance magnitudes, and system dynamics determine the minimum necessary control authority.

IV. Simulation

To illustrate the performance and robustness of the proposed control scheme we provide simulation results on a planar two–link robot manipulator [29], where input saturation is introduced. The system is modeled in the form (1) with $x = [x_1^1, x_1^2, x_2^1, x_2^2]^T$ and the following vectors and matrices:

$$G(t,x) := \begin{bmatrix} p_1 + p_2 + 2p_3 \cos x_2^1 & p_2 + p_3 \cos x_2^1 \\ p_2 + p_3 \cos x_2^1 & p_2 \end{bmatrix}^{-1},$$

$$f(t,x) := \begin{bmatrix} -p_3 \sin x_2^1 x_2^2 & -p_3 \sin x_2^1 (x_1^2 + x_2^2) \\ p_3 \sin x_2^1 x_1^2 & 0 \end{bmatrix} \begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} + \begin{bmatrix} p_4 g \cos x_1^1 + p_5 g \cos(x_1^1 + x_2^1) \\ p_5 g \cos(x_1^1 + x_2^1) \end{bmatrix} + \begin{bmatrix} \sin(p_6 t) \\ \cos(p_6 t) \end{bmatrix},$$

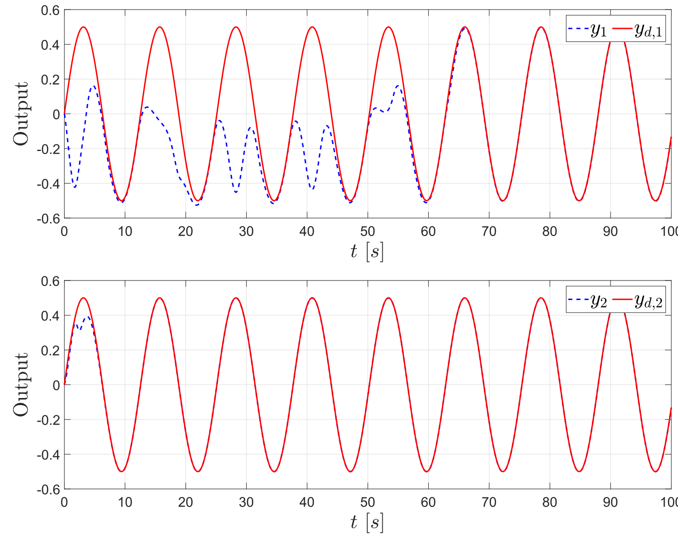

Fig. 1. The outputs of the system.

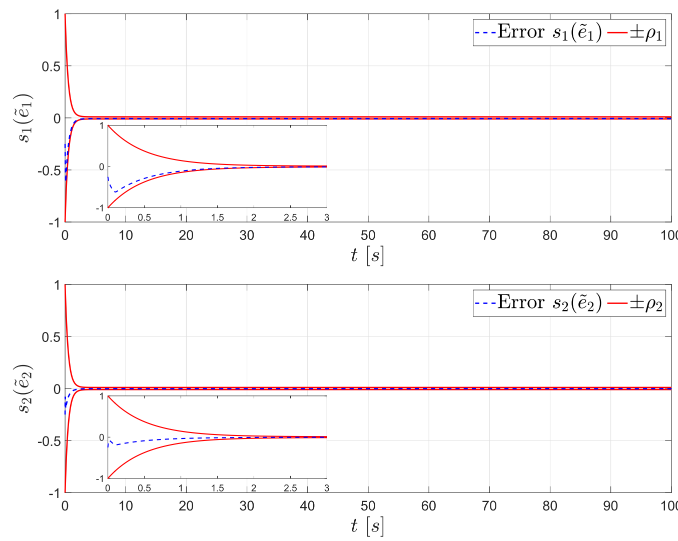

Fig. 2. The filtered errors.

where the second term of f represents gravity and the third bounded exogenous disturbances, with $g := 9.81\ \text{m/s}^2$ and parameter vector:

$$[p_1, p_2, p_3, p_4, p_5, p_6] := [2.9,\ 0.76,\ 0.87,\ 3.04,\ 0.87,\ 0.05].$$

We selected the function $T(\cdot) = \ln\frac{1+(\cdot)}{1-(\cdot)}$ and the actuator saturation limits are set as $u_{1,\max} = 38, u_{2,\max} = 10$.

We adopt performance functions $\rho_i(t)$ as in (3), with $\lambda_i = 2, \rho_i^0 = 1, \rho_i^\infty = 0.01$, for all $i = 1, 2$, and employ the control law (8) with $k_i = 20$, for all $i = 1, 2$, along with the modifier dynamics (5) with $\beta = 1$ and filter (4) with $q_1 = 2.5$. We choose initial conditions $q(0) = \dot{q}(0) = 0$, and a reference $y_d(t) = [0.5 \sin(0.5t)\ 0.5 \sin(0.5t)]^T$.

The results are depicted in Figs. 1–4. Prescribed performance tracking is achieved under input saturation validating the results of Theorem 1. From Fig. 2 we can observe that the prescribed performance envelopes are respected for the filtered errors. Figs. 1, 3 and 4 illustrate the way the proposed control scheme works. When the magnitude of the requested control

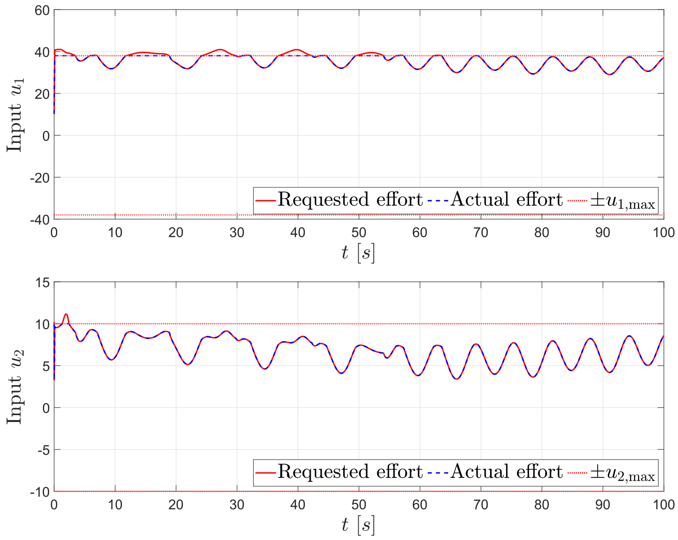

Fig. 3. The requested and actual inputs.

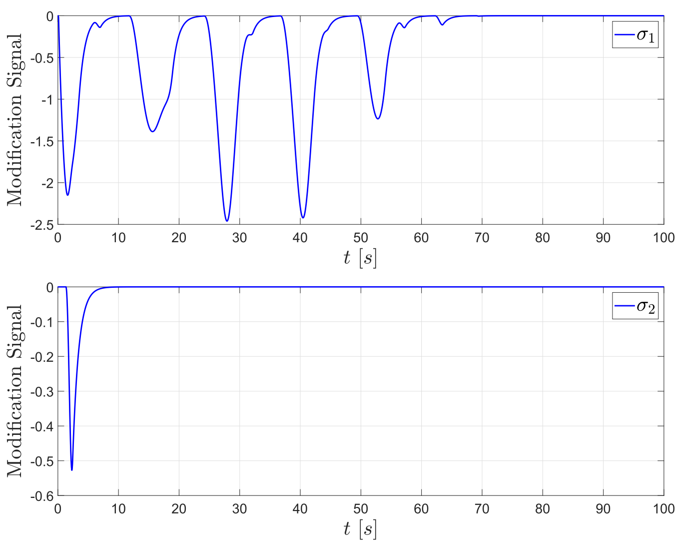

Fig. 4. The modification signals.

effort is larger than the saturation level, the modification mechanism starts to work, allowing the filtered error to relax, thus ensuring stability. Once the requested control effort returns within the saturation levels, the modification signal decays to zero, and the filter allows tracking in the classical prescribed performance control sense.

References

[1] C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance," *IEEE Trans. Autom. Control*, vol. 53, no. 9, pp. 2090–2099, Oct. 2008.

[2] A. Ilchmann, E. P. Ryan, and C. J. Sangwin, "Tracking with prescribed transient behaviour," *ESAIM: Control, Optim. Calculus Variat.*, vol. 7, pp. 471–493, Jan. 2002.

[3] S. P. Karason and A. M. Annaswamy, "Adaptive control in the presence of input constraints," in *Proc. Amer. Control Conf.*, Jun. 1993, pp. 1370–1374.

[4] M. Chen, S. S. Ge, and B. Ren, "Adaptive tracking control of uncertain MIMO nonlinear systems with input constraints," *Automatica*, vol. 47, no. 3, pp. 452–465, Mar. 2011.

[5] C. Chen, G. Zhu, Q. Zhang, and J. Zhang, "Robust adaptive finite-time tracking control for uncertain Euler–Lagrange systems with input saturation," *IEEE Access*, vol. 8, pp. 187605–187614, 2020.

[6] C. Wang and T. Kuang, "Neuroadaptive control for uncertain Euler–Lagrange systems with input and output constraints," *IEEE Access*, vol. 9, pp. 51940–51949, 2021.

[7] K. Shao, R. Tang, F. Xu, X. Wang, and J. Zheng, "Adaptive sliding mode control for uncertain Euler–Lagrange systems with input saturation," *J. Franklin Inst.*, vol. 358, no. 16, pp. 8356–8376, Oct. 2021.

[8] A. Ilchmann and S. Trenn, "Input constrained funnel control with applications to chemical reactor models," *Syst. Control Lett.*, vol. 53, no. 5, pp. 361–375, Dec. 2004.

[9] J. Hu, S. Trenn, and X. Zhu, "Funnel control for relative degree one nonlinear systems with input saturation," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2022, pp. 227–232.

[10] N. Hopfe, A. Ilchmann, and E. P. Ryan, "Funnel control with saturation: Linear MIMO systems," *IEEE Trans. Autom. Control*, vol. 55, no. 2, pp. 532–538, Feb. 2010.

[11] D. Liberzon and S. Trenn, "The bang-bang funnel controller," in *Proc. 49th IEEE Conf. Decis. Control (CDC)*, Dec. 2010, pp. 690–695.

[12] D. Liberzon and S. Trenn, "The bang-bang funnel controller for uncertain nonlinear systems with arbitrary relative degree," *IEEE Trans. Autom. Control*, vol. 58, no. 12, pp. 3126–3141, Dec. 2013.

[13] Y. Wang, J. Hu, J. Li, and B. Liu, "Improved prescribed performance control for nonaffine pure-feedback systems with input saturation," *Int. J. Robust Nonlinear Control*, vol. 29, no. 6, pp. 1769–1788, Apr. 2019.

[14] R. Ji, D. Li, J. Ma, and S. S. Ge, "Saturation-tolerant prescribed control of MIMO systems with unknown control directions," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 12, pp. 5116–5127, Dec. 2022.

[15] R. Ji, D. Li, and S. S. Ge, "Saturation-tolerant prescribed control for nonlinear time-delay systems," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 8, pp. 2495–2508, Aug. 2023.

[16] P. K. Mishra and P. Jagtap, "Approximation-free prescribed performance control with prescribed input constraints," *IEEE Control Syst. Lett.*, vol. 7, pp. 1261–1266, 2023.

[17] K. Yong, M. Chen, Y. Shi, and Q. Wu, "Flexible performance-based robust control for a class of nonlinear systems with input saturation," *Automatica*, vol. 122, Dec. 2020, Art. no. 109268.

[18] P. S. Trakas and C. P. Bechlioulis, "Robust adaptive prescribed performance control for unknown nonlinear systems with input amplitude and rate constraints," *IEEE Control Syst. Lett.*, vol. 7, pp. 1801–1806, 2023.

[19] P. S. Trakas and C. P. Bechlioulis, "Adaptive performance control for input constrained MIMO nonlinear systems," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 54, no. 12, pp. 7733–7745, Dec. 2024.

[20] A. K. Gkesoulis, P. A. Georgakis, G. C. Karras, and C. P. Bechlioulis, "Prescribed performance control for uncertain Euler–Lagrange systems with constrained inputs via virtual-only reference modification," in *Proc. Eur. Control Conf. (ECC)*, Jun. 2025, pp. 404–409.

[21] A. K. Gkesoulis and C. P. Bechlioulis, "A low-complexity adaptive performance control scheme for unknown nonlinear systems subject to input saturation," *IEEE Control Syst. Lett.*, vol. 9, pp. 2115–2120, 2025.

[22] T. Berger, "Input-constrained funnel control of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 69, no. 8, pp. 5368–5382, Aug. 2024.

[23] L. N. Bikas and G. A. Rovithakis, "Prescribed performance under input saturation for uncertain strict-feedback systems: A switching control approach," *Automatica*, vol. 165, Jul. 2024, Art. no. 111663.

[24] S. Li and Z. Xiang, "Adaptive prescribed performance control for switched nonlinear systems with input saturation," *Int. J. Syst. Sci.*, vol. 49, no. 1, pp. 113–123, Jan. 2018.

[25] C. Cheng, Y. Zhang, and S. Liu, "Neural observer-based adaptive prescribed performance control for uncertain nonlinear systems with input saturation," *Neurocomputing*, vol. 370, pp. 94–103, Dec. 2019.

[26] F. Fotiadis and G. A. Rovithakis, "Input-constrained prescribed performance control for high-order MIMO uncertain nonlinear systems via reference modification," *IEEE Trans. Autom. Control*, vol. 69, no. 5, pp. 3301–3308, May 2024.

[27] D. Li, H. Yu, K. P. Tee, Y. Wu, S. S. Ge, and T. H. Lee, "On time-synchronized stability and control," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 52, no. 4, pp. 2450–2463, Apr. 2022.

[28] D. Li, S. S. Ge, and T. H. Lee, "Simultaneous arrival to origin convergence: Sliding-mode control through the norm-normalized sign function," *IEEE Trans. Autom. Control*, vol. 67, no. 4, pp. 1966–1972, Apr. 2022.

[29] K. Zhao, Y. Song, T. Ma, and L. He, "Prescribed performance control of uncertain Euler–Lagrange systems subject to full-state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 8, pp. 3478–3489, Aug. 2018.